COMMUTATIVITY OF NEAR-RINGS WITH DERIVATIONS SATISFYING CERTAIN DIFFERENTIAL IDENTITIES

M.Sc. PROJECT

DEMEKE DANA

FEBRUARY 2019

Haramaya University, Haramaya

Commutativity of Near-Rings with Derivations Satisfying Certain Differential Identities

A Project Submitted to Department of Mathematics,

Postgraduate Program Directorate

HARAMAYA UNIVERSITY

In Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE IN MATHEMATICS

(Algebra)

Demeke Dana

February 2019

Haramaya University, Haramaya

Postgraduate Program Directorate

I hereby certify that I have read and evaluated this Project titled *'Commutativitiy of near ring with derivations satisfying certain differential identities'* prepared under my guidance by Demeke Dana. All feedback given to the student has been incorporated in the project. Therefore, I recommend that it be submitted as fulfilling the project requirement.

Phool Miyan (PhD)	_____	_____
Major Advisor	Signature	Date
Getinet Alemayehu (PhD)	_____	_____
Co-Advisor	Signature	Date

As member of the board of Examiners of the MSc Project Open Defense Examination, we certify that we have read and evaluated the Project prepared by Demeke Dana and examined the candidate. We recommend that the Project be accepted as fulfilling the Project requirement for the degree of *Master of Science in Mathematics* (Algebra).

Seleshi Demie (PhD)	_____	_____
Chairperson	Signature	Date
Melisew Tefera (PhD)	_____	_____
Internal Examiner	Signature	Date
Yibeltal Yitayew (PhD)	_____	_____
External Examiner	Signature	Date

DEDICATION

I dedicate this project manuscript to my father Dana Chemiso and my mother Aynalem Bangejo for their sacrifice throughout my education and support for the success of my life.

STATEMENT OF THE AUTHOR

By my signature below, I declare that this project is my own work. I have followed all ethical and technical principles of scholarship in the preparation, and compilation of this project. Any scholarly matter that is included in the project has been given recognition through citation.

This project is submitted in partial fulfillment of the requirements for Master of Science in Mathematics at Haramaya University. The project is deposited in the Haramaya University Library and is made available to borrowers under the rules of the Library. I solemnly declare that this project has not been submitted to any other institution anywhere for the award of any academic degree, diploma or certificate.

Brief quotations from this project may be made without special permission provided that accurate and complete acknowledgement of the source is made. Requests for permission for extended quotations from or reproduction of this project in whole or in part may be granted by the head of the department or Director of Postgraduate Program Directorate when in his or her judgment the proper use of the material is in the interest of scholarship. In all other instances, however, permission must be obtained from the author of the project.

Name: Demeke Dana

Date: _____

Department: Mathematics

Signature _____

BIOGRAPHICAL SKETCH

The author was born in August 28, 1991 in South Nations Nationality of People Regional State, Wolaita Zone, Boloso Bombe Woreda, Ambe kebele from his mother Aynalem Bangejo and father Dana Chemiso. He attended his primary education at Bombe primary schools. Then after, he joined Areka secondary and preparatory school to attend his secondary education. He joined Wolaita Sodo University and graduated with Bachelor of Science in Mathematics on July, 2015 and then after he worked for in the school in SNNP Regional State, Wolaita zone, Boloso Bombe Woreda, Bombe secondary and preparatory school. After that, he joined the School of Graduate Studies of Haramaya University in September 2017 for M.Sc. study in Mathematics in specialization of Algebra.

ACKNOWLEDGMENTS

I am really thankful to the almighty God whose blessing, protection and kindness helped me to overcome all the challenges and hardships that I faced in the completion of this project. I would like to thank my project major advisor **Dr.Phool Miyan** and co-advisor **Dr.Getinet Alemayehu** for their whole hearted professional advices, guidance by giving constructive comments and useful suggestions. Also thanks to my course instructors and Haramaya University Mathematics department who gave me encouragement and helped me during this study. I would also like to say thanks to my parents, family members, colleagues and friends who encouraged me in various aspects for their untiring support, prayers during this project work and their love and Finally, I would like to say thanks Boloso Bombe Woreda administration education office for give this chance to my graduate study.

NOTATIONS

d Derivation on near ring

R Ring

$Z(N)$ Multiplicative center of near ring

$\zeta(N)$ Additive center of near ring

Table of Contents

Commutativity of Near-Rings with Derivation satisfying certain Differential Identities

ABSTRACT

The aim of this project was to elaborate the conditions for near ring with derivations to be a commutative ring. This is because near ring did not satisfies all the properties of ring. This project studied the conditions under which a near ring becomes a commutative ring. Some conditions imposed certain derivation on near rings satisfying certain differential identities were employed to prove certain commutativity theorems. Important notions, basic definitions, examples, remarks, basic lemmas and theorems related to our study which are required for the development of the main results of this project are presented. The concepts of derivations on near ring and its generalizations were discussed to make idea clear. However, the most important among them was near ring with derivations satisfying certain differential identities. The commutativity of addition and multiplication of near rings satisfying some conditions involving derivations was obtained. Moreover, examples proving the necessity of the primeness condition were given. This project gives a leisurely introduction to the theory of near-rings and its main results suitable for readers with a little knowledge of commutativity of near-ring.

***Keywords*:** Prime Near-Rings, Derivations, Commutativity

1. INTRODUCTION

1.1. Background of the Study

Near rings have been the goal of study for many researchers in Mathematics, since twenties century. A near ring is exactly what is needed to describe the structure of the endomorphism of various mathematical structures adequately. Near rings are generalizations of ring. It is natural to generalize various concepts of rings to near rings.

The study and research on near-rings is very systematic and continuous. Beidleman (1965), Clay (1964), Bell (1971) and Ligh and Luh (1976) have generalized various concepts of near rings. Due to non-ring character of a near ring the results have their own beauty.

Many parts of the well-established theory of rings were transferred to near-rings and near-ring specific features were discovered, building up a theory of near-rings step by step.

The concept of near-ring first introduced by Pilz (1983) as a near ring is a non-empty set N with two binary operations $(N, +, \cdot)$ in the first operation N group (not necessary abelian), in the second operation N semi-group and one of the distributive laws holds.

The study of derivation in rings got impetus soon after Posner (1957) and Herstein (1957) obtained some remarkable results particularly for prime rings. There studies started attracting a wide circle of algebraists in the later part of the 20th century. It plays a significant role in the algebraic geometry and differential algebras. They defined a derivation d on ring R as an additive mapping $d: R \to R$ such that $d(ab) = ad(b) + d(a)b$ for all $a, b \in R$. Then it is said to be derivation on rings.

Several authors like Herstein (1978), Vukman (1989) were studied the derivation of rings in several ways and studied the properties of the derivation of rings.

Motivated by successful results of derivation in rings, the concept of derivation on near ring N was introduced by Bell and Mason (1987) and they defined a derivation d on N to be an additive endomorphism satisfying the "product rule'' such that

$$d(xy) = xd(y) + d(x)y \text{ for all } x, y \in N \qquad (1.1)$$

They extended some results proved by Herstein (1978) derivation on ring to derivation on near rings. Wang in (1994) proved that the second condition of the derivation on near ring is equivalent to equation (1.1) written as follows

$$d(xy) = d(x)y + xd(y) \text{ for all } x, y \in N.$$

Consequently several authors Daif and Bell (1992), Bell (1997), Beidar, *et al*., (1996), Ashraf and Rehman (2002) are generalize many results of derivation of rings to derivation of near-rings.

Boua and Oukhtite (2011) extended some identities proved by Daif and Bell (1992) on derivation of rings to derivation on near-rings which facilitates near rings to be commutative ring.

Bell, *et al*., (2012) investigated derivation on near ring satisfying certain differential identities are commutative ring and they proved some conditions under which near-rings are commutative rings.

Recently, Ashraf, *et al*., (2014) extended some results proved by Bell and Daif (1994) and Ashraf, *et al*., proved near-rings with derivation satisfying certain differential identities are commutative rings.

Motivated by the results of Yhun Shang (2011) very recently, Khan and Madugu (2017) extended some results on prime near-ring with nonzero derivation satisfying some differential identities to show the near rings are commutative ring. They proved if there exist is non-negative integers in the derivation of near rings, in such a way that N admits a non-zero derivation d satisfying some conditions, then near rings are commutative rings.

1.2. Statement of the Problem

Near-rings contain the number of theorems asserting that certain conditions imply commutativity in rings in special classes of near-rings. Boua and Oukhtite (2011) investigated the conditions for which left near ring with derivation to be commutative ring. In the case of commutativity condition in near-rings, one of the most effective conditions in derivation of

near ring is the condition that satisfying certain differential identities on near- rings by (Bell, *et al.*, 2012). They investigated derivations satisfying certain differential identities on a zero symmetric left prime near-ring. After that they proved N be a prime near-ring admits a nonzero derivation d such that

(i) $[d(x), y] = [x, d(y)]$ for all $x, y \in N$

(ii) $[d(x), y] = [d(x), d(y)]$ for all $x, y \in N$

If this conditions of hold, then N is commutative ring.

Motivated by their results, Ashraf, *et al.*, (2014) proved that a zero-symmetric right prime near-ring satisfying certain differential identities are commutative ring and they proved the following properties.

(i) $d([x, y]) \in Z(N)$ for all $x, y \in N$

(ii) $d(xoy) \in Z(N)$ for all $x, y \in N$, the properties hold, then N is commutative ring. They proved a prime near-ring which admits a nonzero derivation d, then the following assertions are equivalent

(i) $d([x, y]) = [d(x), y]$ for all $x, y \in N$

(ii) $[d(x), y] = [x, y]$ for all $x, y \in N$

(iii) N is a commutative ring.

Khan and Madugu in (2017) proved that, if a prime near ring N and there exist non-negative integers in a non-zero derivation d, N is a commutative ring. This project tried to discuss differential identities on near-rings with derivations which facilitate near-ring to be commutative ring.

1.3. Objectives

The main objective of this project was to discuss differential identities on near-rings with derivations which facilitate prime near-rings to be commutative ring.

The study explored the following specific objectives:

- To elaborate the proof of theorems and concepts of derivation on near-rings satisfying certain identities are commutative ring.
- To describe the necessity of primeness condition on derivations in near rings.

- To show commutativity of near rings by using primeness and 2-torsion freeness conditions on derivation in prime near rings.
- To discuss centralizing derivation on near rings satisfying certain differential identities.

2. REVIEW OF RELATED LITERATURE

2.1. Derivation on Near-Rings

Derivation on near-ring is a tool of differential Algebra and the objective of ring theory. There has been considerable interest of investigating commutative condition on derivation in near-rings which is the part of ring theory, more after that of prime near-rings admitting this mapping which are commuting on a prime near-ring with derivation. Being important ring theory, these results are one of the sources of the development on derivation of near-rings.

Bell and Mason (1987) investigated that a near ring contains number of theorems asserting the certain conditions implying commutativity in rings implies multiplicative or additive commutativity in special classes of near rings.They introduced a derivation on near ring N to be an additive endomorphism satisfying the product rule discussed in equation (1.1) and obtained various results regarding the behavior of prime near-ring N as a commutative ring.

Wang (1994) proved the equivalence of derivation on near rings defined in equation (1.1) with $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$.

Biadar and Wang (1996) extended (Poisener and Heristien, 1957) theorems for derivation of prime rings to derivation of prime near-rings. Thereafter they generalize many results of derivation of rings to derivation of near-rings.

Deng *et al.*, (1998) proved that if N is an n-torsion free prime near-ring admitting a derivation d such that $d(N) \subseteq Z(N)$, then either $d\big(Z(N)\big) = 0$ or N is a commutative ring.

Argac and Bell (2004) proved some results on derivation in near-rings and they defined N is zero-symmetric near-ring, then an additive map $d: N \to N$ is derivation d is the same as above equation (1.1). Then additive map on d is to be act as a derivation on the subset S of N. Then equation (1.1) holds for $a, b \in S$. They studied three conditions of near rings that are derivations, product of derivations and commutativity in near-rings and they discussed conditions involving single derivation d or a pair of derivation which force $(N, +)$ to be abelian or N to be commutative ring.

2.2. Commutativity of Derivation on Prime Near -Rings

Near-rings are one of the generalize structures of rings and the study of commutativity of prime near-rings by using derivations was initiated by Bell and Mason (1987). They obtained some basic properties of derivations in near ring. Some of their results, which deal with conditions on derivations, extend earlier commutativity results involving similar conditions on derivations.

Bell and Mason (1987) defined centralizing derivation of a near ring. Inspired by the study of centralizing derivation in rings, they studied the concepts of centralizing derivations in near-rings. A derivation $d: N \to N$ is said to be centralizing (resp. commuting) on a near ring N, if $[x, d(x)] \in Z(N)$ (respectively $[d(x), x] = 0$ holds for all $x, y \in Z(N)$.

Bell and Mason (1992) defined commuting conditions on derivation of a near-ring. They studied the concepts of commuting derivations in near-rings. Further in the mentioned works (Bell, 1997) proved the result for a nonzero semigroup ideal of a near-ring which extends some results to (Bell and Mason, 1994).

The recent literature contains numerous results on commutativity in prime near-rings admitting suitably constrained derivations and several authors have proved comparable results on near-rings satisfying certain differential identities.

Boua and Oukhtite (2011) investigated some results of commutativity of derivation in zero symmetric left near-ring and moreover, they used examples proving the necessity of the primeness condition is given to show near-rings are commutative ring and near-ring with derivation more precisely, proved that a prime near-ring admits a nonzero derivation satisfying certain differential identities must be commutative ring. That is a prime near ring N admits a nonzero derivation d such that in equation (1.2) is satisfied for all $x, y \in N$, then N is commutative ring.

The investigation on certain differential identities on zero symmetric left near ring with derivation is continued by (Bell, *et al*., 2012). Thereafter in the study they proved a 3-prime near rings with derivation satisfying certain differential identities which shows commutativity

near ring. Instead of this N is a 2-torsion free 3-prime near ring and N admits a nonzero derivation d such equation (1.2) are satisfied for all $x, y \in N$, then N is commutative ring.

Boua (2012) studied some conditions under which derivation on zero symmetric left prime near-rings are commutative ring and recently, there has been a great deal of work concerning differential identities on prime near-ring with derivation.

Ghowsi (2014) studied near rings are one of the generalized structures of rings and research done on near-rings is very systematic and continuous behavior or commutative ring. They proved some new commutativity theorems for near–rings with unity have been obtained in their study.

Ashraf, *et al*., (2014) have investigated commutativity of right prime near-ring with derivations satisfying certain differential identities are commutative ring and they proved the equivalent conditions of a 2-torsion free prime near-ring which admits a nonzero derivation d and N is commutative ring.

Recently, Khan and Madugu (2017) proved that a right prime near ring is considered to be commutative ring, if there exist a non-negative integer on near ring N which admits nonzero derivation d satisfying certain differential identities which facilitates a zero symmetric prime near ring N is commutative ring. They used N is a prime near ring admits a non-zero derivation d such that $d(N) \subseteq Z(N)$ show that N is a commutative ring.

3. MATERIALS AND METHODS

This chapter outlines the methods were used to derivation on near-rings and materials used in the study. Sources in the web and libraries were used to collect all the pieces of information about commutativity of near-ring and recorded subsequently. Specifically,

- Relevant journals, books and other project reports were addressed to gather information about the derivation on near rings.
- The collected information was analyzed and arranged keeping coherence.
- Important concepts, definitions, lemmas, examples and theorems were discussed to make ideas clear.
- Important lemmas were used to proof theorems.

4. PRELIMINARY

In this chapter we discuss the concepts of near-ring and illustrate it with examples. Several of its basic properties are given and most of the results given here are from the book of Pilz in (1983). Moreover, this chapter contains basic definitions and fundamental results in the theory of near rings needed for the development of the project in the subsequent sections of the present chapter.

4.1. Some Theoretic Concepts of Near-Rings

Definition 4.1.1(Pilz, 1983) A near-ring is a set N together with two binary operations '$+$' and '$\cdot$' such that

(i) $(N, +)$ is a group (not necessarily abelian)

(ii) $(N, \cdot)$ is a semi group (under multiplication)

(iii) $x(y+z) = xy + xz$ for all $x, y, z \in N$ (left distributive law).

This near-ring will be termed as left near-ring. If $(y+z)x = yx + zx$, for all $x, y, z \in N$ instead of condition (iii), then we call N is a right near-ring.

In this project we use the word near ring to mean left near ring, unless otherwise we specified. As in both the cases, the theory of near-rings runs completely parallel, we may consider left near-rings throughout and for simplicity call them as near-rings.

Example 4.1(Pilz,1983) The most natural example of right near-ring is given by the set N of identity preserving maps of an additive group G (not necessarily abelian) into itself with point wise addition and composition of mappings as multiplication.

$$(f+g)(x) = f(x) + g(x)$$
$$(fg)(x) = f(g(x)) \text{ for all } f, g \in N, x \in G.$$

Verification: Let f is identity preserving and defined by

$N = \{f(G,+) \to (G,+), where\ f\ is\ identity\ preserving\}$.

Therefore, $f(0) = 0$.

We show that point wise addition is group.

Closure property: For all $f, g \in N(G)$ and $x \in G$ implies that $f + g \in N(G)$,

$$(f+g)(x)=0.$$

And $(f\cdot g)(x)=f(g(x))=0$ for all $f,g\in N(G)$ and $x\in G$. Therefore $(f\cdot g)(x)\in N(G)$.

This show it satisfies all the properties near ring, since $(N,+)$ is group and $(N,\cdot)$ is semi group.

We check distributive property for all $f,g,h\in N(G)$. Then

$\big((f+g)\cdot h\big)(x)=(f\cdot h)(x)+(g\cdot h)(x)$ for all $f,g\in N(G)$, $x\in G$. Therefore right distributive law holds in $N(G)$, N is a right near-ring.

Definition 4.1.2 (Boua and Oukhtite, 2011) An element x of a near ring N is called distributive if $(y+z)x=yx+zx$ for all elements $x,y,z\in N$, then x is said to be a left distributive element.

Remark 4.1.2 For a left near-ring $(N,+,\cdot)$, if zero is the additive identity of $(N,+,\cdot)$, then $x0=0$ and $x(-y)=-(xy)$ for all $x,y\in N$ and a near-ring N is distbutive if each of its element is distributive element.

It is clear that $x0=0$ for all x in N, now we have $x(-y)=-(xy)$ for all $x,y\in N$. Since $x(-y)+(xy)=x(-y+y)=x0=0$ for all $x\in N$.

Example 4.2. Let $N=\{0,a,b,c,x,y\}$ with addition $(+)$ and multiplication $(\cdot)$ tables defined as follows:

Addition

$+$	0	a	b	c	x	y
0	0	a	b	c	x	y
a	a	0	y	x	c	b
b	b	x	0	y	a	c
c	c	y	x	0	b	b
x	x	b	c	a	y	0
y	y	c	a	b	0	x

Multiplication

$\cdot$	0	a	b	c	x	y
0	0	0	0	0	0	0
a	0	a	a	a	0	0
b	0	a	a	a	0	0
c	0	a	a	a	0	0
x	0	0	0	0	0	0
y	0	0	0	0	0	0

From the two tables wget $(N, +, \cdot)$ is left near-ring. Now we can check $(a + b)x = yx = 0$ and now $ax + bx = 0 + 0 = 0$. similarly we can check for all elements right distributive properties. Then in N every element is distributive, N is distributive near-ring.

Definition 4.1.3 (Boua and Oukhtite, 2011) A near-ring N is said to be zero-symmetric near ring if $x0 = 0 = 0x$ for all $x \in N$. If $0x = 0$, then N is said to be zero-symmetric left near-ring.

In this project we use zero symmetric near ring.

Example 4.3. Let $N = \{0,1,2,3\}$ with addition and multiplication tables defined as below:

Addition

$+$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Multiplication

$\cdot$	0	1	2	3
0	0	0	0	0
1	0	3	2	1
2	0	0	0	0

3	0	1	2	3

It can be verified that $(N, +, \cdot)$ is a zero-symmetric near ring.

Definition 4.1.4. The least positive integer n (if exists) such that $nx = 0$ for all $x \in N$ is called the characteristic of the near ring N, which is generally expressed as $char(N) = n$. If no such positive integer exists, then N is said to have characteristic zero.

Definition 4.1.5 (Boua and Oukhtite, 2011) An element x in a near-ring N is said to be 2-torsion free if $2x = 0$ imples that $x = 0$ for every $x \in N$.

Definition 4.1.6 (Gerhand, 2009) Let Z_r zero divisors of near ring and right zero divisors is defined by $Z_r = \{n \in N/xn = 0 \; for \; some \; x \in N/\{0\}\}$ and left zero divisors Z_l of a near-ring by $Z_l = \{n \in N/nx = 0 \; for \; some \; x \in N/\{0\}\}$.

Definition 4.1.7. A near-ring N is said to be zero- ommutative if $xy = 0$ implies $yx = 0$ for all $x, y \in N$.

Example 4.4. Let $N = \{0, a, b, c\}$ with addition and multiplication table difined as below

Addition

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	a

Multiplication

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	0	0
c	0	c	0	0

We check that $ab = 0$, $ba = 0$ and $bc = 0$, $cb = 0$. Then $ab = 0$ implies $ba = 0$ and $bc = 0$ implies that $cb = 0$. Thus we get if $xy = 0$ implies $yx = 0$ for all $x, y \in N$. Hence $(N, +, \cdot)$ is a zero commutative near ring.

Definition 4.1.8. Let $(N_1, +, \cdot)$ and $(N_2, +, \cdot)$ be two near rings. Then a mapping $f: N_1 \to N_2$ is called a near ring homomorphism if

(i) $f(r_1 + r_2) = f(r_1) + f(r_2)$,

(ii) $f(r_1 \cdot r_2) = f(r_1) \cdot f(r_2)$ for all $r_1, r_2 \in N_1$

Remark 4.1.7. Image of a near-ring under a near-ring homomorphism is again a near ring.

Definition 4.1.9. Let $(N, +)$ be group and multiplication defined by $xy = y$ for all $x, y \in N$, then N is constant near-ring.

Definition 4.1.10 (Bell and Mason, 1987) A near-ring N is said to be prime if and only if for any $x, y \in N$, $xNy = \{0\}$ implies that $x = 0$ or $y = 0$.

Definition 4.1.11 (Bell and Mason, 1987) An additive center of a near-ring N is the set of all those elements of N which commute with every element of N under addition and defined as

$$\zeta(N) = \{z \in N / \, z + x = x + z \; for \; all \; x \in N\}$$

Definition 4.1.12 (Boua and Oukhtite, 2011) The multiplicative center of a near-ring N is the set of all those elements of N, which commutes with every element of N under multiplication and it is denoted by $Z(N)$, that is

$$Z(N) = \{x \in N \mid \; xy = yx \text{ for all } y \in N\}.$$

Definition 4.1.13 (Boua and Oukhtite, 2011) For any pair of elements x, y in a near ring N, additive commutator is denoted by $(x, y) = x + y - x - y$, multiplicative commutator denoted by $[x, y] = xy - yx$ and anti-commutator is denoted by $xoy = xy + yx$.

Properties of commutators: Let N be a near ring, then the following properties are satisfied.

(i) $[x, yx] = [x, y]x$,

(ii) $xo(yx) = (xoy)x$,

(iii) $[xy, y] = [x, y]y$,

(iv) $(xoy)oy = (xoy)y$,

(v) $\big[[x, y], z\big] + \big[[y, z], x\big] + \big[[z, x], y\big] = 0$, for all $x, y \in N$.

Let we show (i) $[x, yx] = [x, y]x$

From the left side

$[x, yx] = x(yx) - (yx)x = xyx - yxx = (xy - yx)x = [x, y]x$ for all $x, y \in N$.

4.2. Some Basic Result in Near-Rings with Derivations

In this section we give some well-known basic lemmas and prove some auxiliary results including examples which will be used in the next sections of the project.

Definition 4.2.1 (Bell and Mason, 1987) An additive mapping $d: N \to N$ is said to be a derivation on a near-ring N. If

(i) $d(x + y) = d(x) + d(y)$,

(ii) $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$.

Example 4.2.1. Consider that $N = N_1 \oplus N_2$, where N_1 is a zero symmetric left near ring and N_2 is commutative ring admitting nonzero derivation δ. Define $d: N \to N$ by $d((u_1, u_2)) = (0, \delta(u_2))$. Then d is a derivation on N and $N = N_1 \oplus N_2$

$N = \{(n_1, n_2) / n_1 \in N_1 \; and \; n_2 \in N_2\}$.

$(N, +)$ is group and $(N, \cdot)$ is semigroup

$(m_1, n_1) + (m_2, n_2) = (m_1 + m_2, \; n_1 + n_2)$ and $(m_1, n_1) + (m_2, n_2) = (m_1 \cdot m_2, \; n_1 \cdot n_2)$ from this we check $(N, +, \cdot)$ is left near ring.

Verification: Given that $\delta: N_2 \to N_2$ is derivation and

$d(u_1, u_2) = (0, \delta(u_2)$ (4.1)

$d(N_1 \oplus N_2) \to N_1 \oplus N_2$ and $d(v_1, v) = (0, \delta(v_2)$

(i) $d((u_1, u_2) + (v_1, v_2)) = d(u_1, u_2) + d(v_1, v_2)$.

(ii) $d((u_1, u_2)(v_1, v_2)) = (u_1, u_2)d(v_1, v_2) + d(u_1, u_2)(v_1, v_2)$ for all (u_1, u_2), $(v_1, v_2) \in N$ and $d: N \to N$.

Let to show that d is derivation on N, since (u_1, u_2), $(v_1, v_2) \in N$

(i) $d((u_1, u_2) + (v_1, v_2)) = d(u_1, u_2) + d(v_1, v_2)$ for all (u_1, u_2), $(v_1, v_2) \in N$.

From the left side,

$d((u_1, u_2)+(v_1, v_2)) = d(u_1 + v_1, u_2 + v_2)$

$= (0, \delta(u_2 + v_2))$

$= (0, \delta(u_2 + v_2))$, by definition of δ.

$= (0 + 0, \delta(u_2) + \delta(v_2))$

$= (0, \delta(u_2))+ (0, \delta(u_2))$ for all $u_2, v_2 \in N$, (4.1)

From the right side,

$d(u_1, u_2) + d(v_1, v_2) = (0, \delta(u_2)) + (0, \delta(v_2))$

$= (0, \delta(u_2)) + (0, \delta(v_2))$ for all $(u_2, v_2) \in N$, by definition of δ. (4.2)

Then from equation (4.1) and (4.2)

$d((u_1, u_2)+(v_1, v_2)) = d(u_1, u_2) + d(v_1, v_2)$ for all (u_1, u_2), $(v_1, v_2) \in N$.

Now to show second properties

$d((u_1, u_2)(v_1, v_2)) = (u_1, u_2)d(v_1, v_2) + d(u_1, u_2)(v_1, v_2)$ for all (u_1, u_2), $(v_1, v_2) \in N$.

Since, by definition of derivation $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$.

From the left side,

$d((u_1, u_2) (v_1, v_2)) = d(u_1.v_1, u_2.v_2))$

$= (0, \delta(u_2. v_2))$ for all $(u_2, v_2) \in N$ (4.3)

From the right side,

$(u_1, u_2)\, d(v_1, v_2) + d(u_1, u_2)(v_1, v_2) = (u_1, u_2) \big(0 , \delta(v_2)\big) + (0, \delta(u_2)) (v_1, v_2)$

$= \big(0 , \delta(u_2 . v_2)\big) + \big(0 , \delta(u_2 . v_2)\big)$

Since N is zero symmetric, we get

$(u_1, u_2)\, d(v_1, v_2) + d(u_1, u_2)(v_1, v_2) = (0, \delta(u_2. v_2))$ for all $(u_2, v_2) \in N$ (4.4)

Hence from equation (4.3) and (4.4), we get

$d((u_1, u_2)(v_1, v_2)) = (u_1, u_2)d(v_1, v_2) + d(u_1, u_2)(v_1, v_2)$ for all (u_1, u_2), $(v_1, v_2) \in N$.

Therefore the two conditions of derivation on near rings are satisfied in (i) and (ii). Thus d is derivation on a near-ring $N = N_1 \oplus N_2$.

It was shown by Wang (1994), the condition (ii) of definition 4.2.1 is equivalent to $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$, which facilitates the study of derivations in near-rings.

The following lemmas (Lemma 4.2.1, Lemma 4.2.2 and Lemma 4.2.3) proved by Bell and Mason (1987) and Wang (1994) which will useful for the proof of next chapter. We prove these lemmas step by step in detail.

The part (i) of the following Lemma 4.2.1 is proved by Bell and Mason (1987) and part (ii) is proved by Wang (1994).

Lemma 4.2.1 (Wang, 1994) Let d be an arbitrary derivation on the near-ring N, then N satisfies the following partial distributive law:

(i) $(xd(y) + d(x)y)z = xd(y)z + d(x)yz$ for all $x, y \in N$

(ii) $\big(d(x)y + xd(y)\big)z = d(x)yz + xd(y)z$ for all $x, y \in N$

Proof (i) We know that $d(xy) = xd(y) + d(x)y$, for all $x, y, z \in N$.

Now, we have

$$d\big((xy)z\big) = xyd(z) + d(xy)z$$
$$= xyd(z) + (xd(y) + d(x)y)z, \text{ for all } x, y, z \in N.$$

Therefore, $d\big(x(yz)\big) = xd(yz) + d(x)yz$

$$= x(yd(z) + d(y)z)) + d(x)yz,$$
$$= xyd(z) + xd(y)z + d(x)yz \text{ for all } x, y, z \in N,$$

Since $d\big((xy)z\big) = d\big(x(yz)\big)$, it follows that

$(xd(y) + d(x)y)z = xd(y)z + d(x)yz$ for all $x, y, z \in N$.

(ii) We know that $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$.

Then, $d\big(x(yz)\big) = d(x)yz + xd(yz)$

$$= d(x)yz + x(d(y)z + yd(z) \text{ for all } x, y, z \in N$$

Therefore

$$d\big(x(yz)\big) = d(x)yz + xd(y)z + xyd(z) \text{ for all } x, y, z \in N \quad (4.5)$$

Now we take $d\big((xy)z\big) = d(xy)z + (xy)d(z)$

$$= \big(d(x)y + xd(y)\big)z + xyd(z) \text{ for all } x, y, z \in N. \quad (4.6)$$

Since $(xy)z = x(yz)$, it follows from equation (4.5) and equation (4.6)

$\big(d(x)y + xd(y)\big)z = d(x)yz + xd(y)z$ for all $x, y, z \in N$.

Lemma 4.2.2 (Bell and Mason, 1987) Let d be a derivation on N and suppose $u \in N$ is not a left zero divisor. If $[u, d(u)] = 0$, then (x, u) is a constant for every $x \in N$.

Proof. Consider $u(u + x) = u^2 + ux$, for all $u, x \in N$

Now,

$$d\big(u(u + x)\big) = d(u^2 + ux) \text{ for all } u, x \in N$$

Then we have

$$d\big(u(u + x)\big) = ud(u + x) + d(u)(u + x)$$
$$= ud(u) + d(u)u + ud(x) + d(u)x \text{ for all } u, x \in N$$

which reduces to

$$ud(x) + ud(u) = d(u)u + ud(x) \text{ for all } u, x \in N,$$

From this we get

$$\big(ud(x) + ud(u) - ud(u) - ud(x)\big) = 0, \text{ for all } u, x \in N,$$

Since, $d(u)u = ud(u)$, equation is expressed as

$$u\big(d(x) + d(u) - d(x) - d(u)\big) = 0 \text{ for all } u, x \in N,$$

$$u\big(d(x), d(u)\big) = 0 \text{ for all } u, x \in N.$$

By using definition of additive group commutator, we have

$$ud\big((x, u)\big) = 0 \text{ for all } u, x \in N.$$

Thus, $d\big((x, u)\big) = 0$.

Lemma 4.2.3 (Bell and Mason, 1987) Let N be a prime near-ring.

(i) If $z \in Z/\{0\}$, then z is not a zero divisor.

(ii) If Z contains a nonzero element z for which $z + z \in Z$, then $(N, +)$ is abelian.

(iii) Let d be a nonzero derivation on N. Then $xd(N) = \{0\}$ implies $x = 0$ and $d(N)x = \{0\}$ implies $x = 0$.

(iv) If N is 2-torsion free and d is a derivation on N such that $d^2 = 0$, then $d = 0$.

Proof: (i) Since $z \in N$, we have

So there is no $x \in N \,/\, \{0\}$ such that $zx = 0$, since z is zero divisor.

$$zNx = \{0\}, \text{ since } N \text{ is prime } x = 0.$$

(ii) Let $z \in Z(N)\backslash\{0\}$ be an element such that $z + z \in Z(N)$ and let $x, y \in N$. Since $z + z$ is distributive we get

$$\begin{aligned}(x + y)(z + z) &= x(z + z) + y(z + z)\\ &= xz + xz + yz + yz\\ &= z(x + y + x + y).\end{aligned}$$

On the other hand,

$$\begin{aligned}(x + y)(z + z) &= (x + y)z + (x + y)z\\ &= z(x + y + x + y)\ x, y \in N,\end{aligned}$$

Thus,

$$x + y + x + y = x + y + x + y \text{ and therefore } x + y = y + x$$

(iii) Let $xd(N) = \{0\}$, and let r, s be arbitrary elements of N.

Then, we have

$$\begin{aligned}0 &= xd(rs)\\ &= xrd(s) + xd(r)s\\ &= xrd(s).\end{aligned}$$

So that

$$xNd(N) = \{0\}$$

Since N is prime, $x = 0$ or $d(N) = 0$. But $d(N) \neq 0$ N is left near ring, assumption which implies that $x = 0$.

(iv) For arbitrary $x, y \in N$, we have $\begin{aligned}0 = d^2(xy) &= d(xd(y) + d(x)y)\\ &= xd^2(y) + d(x)d(y) + d(x)d(y) + d^2(x)y\\ &= 2d(x)d(y),\end{aligned}$

Since N is 2-torsion free, we get $d(x)d(N) = \{0\}$ for each $x \in N$, then (iii) yields $d = 0$.

The following Theorem 4.2.1 shows the equivalence of both conditions on derivations in near rings.

Theorem 4.2.1 (Wang, 1994) A derivation d on a near ring N is a derivation if and only if $d(xy) = d(x)y + xd(y)$, for all $x, y \in N$.

Proof. Suppose d is a derivation, thus by definition 4.2.1

$$d(xy) = xd(y) + d(x)y, \text{ for all } x, y \in N,$$

Since, we have the properties of near ring

$x(y + y) = xy + xy$, it follows that the derivation on near ring N,

Let $d\big(x(y + y)\big) = xd(y + y) + d(x)(y + y)$

$$= xd(y) + xd(y) + d(x)y + d(x)y \text{ for all } x, y \in N. \quad (4.7)$$

Now, d is an endomorphism

$$d(xy + xy) = d(xy) + d(xy)$$

$$d(xy) + d(xy) = xd(y) + d(x)y + xd(y) + d(x)y \text{ for all } x, y \in N. \quad (4.8)$$

From equation (4.7) and (4.8) we get

$$xd(y) + d(x)y = d(x)y + xd(y), \text{ for all } x, y \in N.$$

Hence,

$$d(xy) = d(x)y + xd(y), \text{ for all } x, y \in N$$

For the converse, suppose that

$$d(xy) = d(x)y + xd(y), \text{ for all } x, y \in N.$$

Since we have $x(y + y) = xy + xy$, for all $x, y \in N$, it follows that

$$d\big(x(y + y)\big) = d(x)(y + y) + xd(y + y)$$

$$= d(x)y + d(x)y + xd(y) + xd(y), \text{ for all } x, y \in N \quad (4.9)$$

Also, $d(xy + xy) = d(xy) + d(xy)$

$$= d(x)y + xd(y) + d(x)y + xd(y), \text{ for all } x, y \in N \quad (4.10)$$

From equation (4.9) and (4.10), we get

$$d(x)y + xd(y) = xd(y) + d(x)y, \text{ for all } x, y \in N.$$

Thus,

$$d(xy) = xd(y) + d(x)y, \text{ for all } x, y \in N.$$

Hence, d is a derivation on N.

The following Theorem 4.2.2 and Theorem 4.2.3 are due to Bell and Mason (1987) which is important on derivation of near-rings which facilitates the commutativity of near ring.

Theorem 4.2.2 (Bell and Mason, 1987) If a prime near-ring N admits a nontrivial derivation d for which $d(N) \subseteq Z(N)$, then $(N, +)$ is abelian. Moreover, if N is 2-torsion-free, then N is a commutative ring.

Proof. Let c be an arbitrary constant and let x be a non constant. Then

$$d(xc) = xd(c) + d(x)c$$
$$= d(x)c \in Z(N)$$

Since $d(x)c \in Z(N) \setminus \{0\}$, it follows that $c \in Z(N)$.

Since $c + c$ is constant for all constants c, it follows from Lemma 4.2.3(ii) that $(N, +)$ is abelian, provides that there exists a nonzero constant.

Assume, that 0 is the only constant. Since d is obviously commuting, it follows from Lemma 4.2.2 that all u which are not zero divisors belong to the center of $(N, +)$, denoted by $\zeta(N)$.

In particular if $x \neq 0$, $d(x) \in \zeta(N)$. But then for all $y \in N$, we get

$$d(y) + d(x) - d(y) - d(x) = d(y + x - y - x)$$
$$= d\big((y, x)\big) = 0, \text{ since } d \text{ is additive}$$

Hence, $(x, y) = 0$.

$$x + y - y - x = 0 \text{ for all } x, y \in N$$
$$x + y = y + x, \text{ for all } x, y \in N$$

Therefore $(N, +)$ is Abelian.

To complete the proof by assuming that N is 2-torsion free and showing that N is commutative. By Lemma 4.2.1, $(xd(y) + d(x)y)z = xd(y)z + d(x)yz$ for all $x, y \in N$ and using the fact of $d(xy) \in Z(N)$, we get

$$zxd(y) + zd(x)y = xd(y)z + d(x)yz \text{ for all } x, y, z \in N.$$

Since $(N, +)$ is abelian and $d(N) \subseteq Z(N)$, this equation is rearranged to yields

$$d(y)[z, x] = d(x)[y, z] \text{ for all } x, y, z \in N.$$

Suppose N is not commutative. Choosing $y, z \in N$ with $[y, z] \neq 0$ and letting $x = d(x)$, we get $d^2(x)[y, z] = 0$ for all $x \in N$ and since the centeral element $d^2(x)$ cannot be nonzero divisor of zero, we conclude that $d^2(x) = 0$ for all $x \in N$. But by Lemma 4.2.3(iv), this cannot happen for nontrivial derivation d.

We consider from Theorem 4.2.3 a commutativity of prime near rings with admitting suitably constrained derivations.

Theorem 4.2.3 (Bell and Mason, 1987) Let N be a prime near-ring admitting a nontrivial derivation d such that $[d(x), d(y)] = 0$ for all $x, y \in N$. Then $(N, +)$ is abelian and if N is 2-torsion-free as well, then N is a commutative ring.

Proof. The argument used in the proof of lemma 4.2.3(ii) shows that if both z and $z + z$ commute elementwise with $d(N)$, then $zd(c) = 0$ for all additive commutators c.

Thus, by taking $z = d(x)$, we get

$$d(x)d(c) = 0 \text{ for all } x \in N$$

So that $d(c) = 0$ by Lemma 4.2.3(iii).

Since wc is also additive commutator for any $w \in N$, we have $d(wc) = 0 = d(w)c$ and Lemma 4.2.3(iii) gives $c = 0$.

Assume now that N is 2-torsion-free. By the partial distributive law,

$$d(d(x)y)d(z) = d(x)d(y)d(z) + d^2(x)yd(z) \text{ for all } x, y, z \in N,$$

Hence, $d^2(x)yd(z) = d(d(x)y)d(z) - d(x)d(y)d(z)$

$$= d(z)(d(dx)y) - d(x)d(y)) \; = \; d(z)d^2(x)y = d^2(x)d(z)y$$

Thus,

$$d^2(x)(yd(z) - d(z)y) = 0 \text{ for all } x, y, z \in N$$

Replacing y by yt, we obtain

$$d^2(x)ytd(z) = d^2(x)d(z)yt \text{ for all } x, y, z, t \in N$$

So that

$$d^2(x)N[t, d(z)] = \{0\} \text{ for all } x, z, t \in N.$$

The primeness of N now shows that either $d^2 = 0$ or $d(N) \subseteq Z(N)$ and since the first of these condition is impossible by lemma 4.2.3(iv), the second must hold and N is a commutative ring by Theorem 4.2.2.

5. DERIVATIONS ON NEAR-RINGS SATISFYING CERTAIN DIFFERENCIAL IDENTITIES

The literature on near-rings contains a number of theorems asserting that certain conditions implying commutativity in near rings imply multiplicative or additive commutativity in special classes of near rings. Some authors Bell and Mason (1987), Boua and Oukhtite (2011), Bell, *et al.*, (2012), Ashraf *et al.*, (2014) and Khan and Madugu (2017) established several commutativity theorems for zero-symmetric prime near-rings admitting a suitably-constrained derivation. But all of the theorems did not completely proved in detail, in this project we elaborate proofs those theorems step by step in clear and detail way. More precisely, we prove that a prime near-ring which admits a nonzero derivation satisfying certain differential identities is commutative ring.

5.1. Differential Identities involving Derivation on Near-Rings

Daif and Bell (1994) established that a prime ring R must be commutative ring, if it admits a derivation d such that either $d([x,y]) = \pm[x,y]$ for all , $y \in K$, where K is a nonzero ideal of ring R. Motivated by their results Boua and Oukhtite (2011) proved some results to show zero symmetric left near-ring with derivation satisfying certain differential identities must be a commutative ring.

Theorems 5.1.1 and Theorem 5.1.2 provides the conditions under which prime near-ring to be a commutative ring. Moreover, examples proving the necessity of the primeness condition are given.

Theorem 5.1.1 (Boua and Oukhtite, 2011) Let N be a prime near-ring. If N admits a nonzero derivation d such that $d([x,y]) = [x,y]$ for all $x, y \in N$, then N is a commutative ring.

Proof. Assume that

$d([x,y]) = [x,y]$ for all $x, y \in N$ (5.1)

Replacing y by xy in equation (5.1), because of $[x,xy] = x[x,y]$, we get $x[x,y] = d(x[x,y])$ for all $x, y \in N$

Since $d(x[x,y]) = xd([x,y]) + d(x)[x,y]$,

Then according to equation (5.1) we obtain

$$x[x,y] = x[x,y] + d(x[x,y]) \text{ and therefore } d(x)[x,y] = 0,$$

Hence,

$d(x)(xy - yx) = 0$ for all $x, y \in N$ (5.2)

Substituting yz for y in equation (5.2), we obtain $d(x)y(xz - zx) = 0$ and the equation (5.2) which leads to

$d(x)N(xz - zx) = \{0\}$ for all $x, z \in N$ (5.3)

Since N is prime, equation (5.3) reduces to

$d(x) = 0$ or $[x, z] = 0$ for all $x, z \in N$ (5.4)

From equation (5.4) it follows that for each fixed $x \in N$ we have

$d(x) = 0$ or $x \in Z(N)$ (5.5)

But $x \in Z(N)$, since $d \neq 0$ on N, then $x \in Z(N)$ implies that $d(x) \in Z(N)$

Then equation (5.5) forces

$d(x) \in Z(N)$, for all $x \in N$ (5.6)

In the light of equation (5.6), we have $d(N) \subseteq Z(N)$ and using equation (5.5), we conclude that N is a commutative ring.

The following example demonstrates that the primeness hypothesis in Theorem 5.1.1 is necessary even in the case of arbitrary rings.

Example 5.1 (Boua and Oukhtite, 2011) Let R be a commutative ring which is not a zero ring and consider $N = \left\{\begin{pmatrix} 0 & 0 \\ x & y \end{pmatrix} / x, y \in R\right\}$, if we define a derivation $d: N \to N$ by $d\begin{pmatrix} 0 & 0 \\ x & y \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}$, then it is straightforward to check that d is a nonzero derivation on a near ring N. On the other hand, if there is $A = \begin{pmatrix} 0 & 0 \\ x_1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 0 & y_1 \end{pmatrix}$ for all $x_1, y_1 \in R$. We have $ANB = \{0\}$ for all $A, B \in N$, but $A \neq 0$ and $B \neq 0$, from this observations N is not prime. Moreover, d satisfies the condition $d([A, B]) = [A, B]$ for all $A, B \in N$. But $A \cdot B \neq B \cdot A$, which yields that N is not commutative ring.

Theorem 5.1.2 (Boua and Oukhtite, 2011) Let N be a prime near-ring. If N admits a nonzero derivation d such that $d(xoy) = xoy$ for all $x, y \in N$, then N is a commutative ring.

Proof. By the hypotheses, we have

$d(xoy) = xy + yx$ for all $x, y \in N$. (5.7)

Replacing y by xy in equation (5.7), we get

$d\big(xo(xy)\big) = x^2y + xyx$ for all $x, y \in N$. (5.8)

Since, we have $xo(xy) = x(xoy)$,

Then equation (5.8) yields $d\big(xo(xy)\big) = d(x(xoy))$

$= xd(xoy) + d(x)xoy$ for all $x, y \in N$,

From the given we have $d(xoy) = xoy$, Hence, equation (5.8) reduces to

$d\big(xo(xy)\big) = x(xoy) + d(x)(xoy) = x^2y + xyx + d(x)(xoy)$ for all $x, y \in N$. (5.9)

As we have from equations (5.9), we get

$$x^2y + xyx = x^2y + xyx + d(x)(xoy)$$

Then equation (5.9) assures that

$d(x)(xoy) = 0$ for all $x, y \in N$. This leads to

$d(x)xy = -d(x)yx$ for all $x, y \in N$ (5.10)

Substituting yz for y in equation (5.10) we find that

$$\begin{aligned} -d(x)yzx &= d(x)xyz \\ &= (-d(x)yx)z \\ &= d(x)y(-x)z \text{ for all } x, y, z \in N \end{aligned}$$

Since from equation (5.10), we get

$-d(x)yzx = d(x)y(-x)z$ and equation (5.10) becomes

$d(x)yz(-x) = d(x)y(-x)z$ for all $x, y, z \in N$ (5.11)

Taking $-x$ instead of x in equation (5.11) gives $d(-x)yzx = d(-x)yxz$ for all $x, y, z \in N$

So that $d(-x)y(zx - xz) = 0$, $x, y, z \in N$

Therefore we get

$d(-x)N[z, x] = 0$ for all $x, z \in N$. (5.12)

By primeness, equation (5.12) assures that for each $x \in N$, either $x \in Z(N)$ or $d(x) = 0$,

Accordingly,

$d(x) = 0$ or $[x, z] = 0$ for all $x, z \in N$. (5.13)

From equation (5.13) it follows that for each $x \in N$, we have

$d(x) = 0$ or $x \in Z(N)$ for all $x \in N$ (5.14)

But $x \in Z(N)$ also implies that $d(x) \in Z(N)$ and equation (5.14) forces

$d(x) \in Z(N)$ for all $x \in N$ (5.15)

In the light of equation (5.15), it follows that $d(N) \subseteq Z(N)$ we conclude that N is a commutative ring.

The following example proves that the primeness hypothesis in Theorem 5.1.2 is necessary even in the case of arbitrary rings.

Example 5.2 (Boua and Oukhtite, 2011) Let S be any ring. Next, let us consider the ring $N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} / x, y, z \in S \right\}$. Define a map $d: N \to N$ such that

$d\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} = \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ for all $x, y \in S$. Then d is a nonzero derivation on N. If we take $A = \begin{pmatrix} 0 & x_1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ with $A \neq 0$ and $B \neq 0$, then $ANB = \{0\}$ proving that N is not prime. Moreover, it can be easily seen that d is a derivation on N and satisfies $d(AoB) = AoB$ for all $A, B \in N$. In the case of primeness hypothesis not satisfied and N is a non commutative ring.

Ashraf *et al.*, (2014) proved the following Theorem 5.1.3, if there exist a zero derivation on a right prime near ring N, then the following condition holds.

T**heorem 5.1.3** (Ashraf *et al.*, 2014) Let N be a 2-torsion free prime right near-ring. Then there exists no nonzero derivation d of N such that $d(x)oy = xoy$ for all $x, y \in N$.

Proof. Suppose d is a nonzero derivation

$d(x)oy = xoy$ for all $x, y \in N$ (5.16)

Replacing x by xy in equation (5.16) we obtain

$d(xy)oy = (xy)oy = (xoy)y = (d(x)oy))y$ for all $x, y \in N$ (5.17)

Since, we have $d(xy)oy = \big(d(x)y + yd(x)\big) = d(xy)y + yd(xy)$,

Then, according to Lemma 4.2.1(ii) we obtain

$$d(xy)oy = (xd(y) + d(x)y)y + y(xd(y) + d(x)y)$$
$$= d(x)y^2 + xd(y)y + yxd(y) + yd(x)y,$$

By using the properties, we get

$$d(x)y^2 + yd(x)y = d(x)y^2 + xd(y) + yxd(y) + yd(x)y$$
$$= yxd(y)xd(y)y = 0, \text{ for all } x, y, z \in N.$$

Therefore,

$yxd(y) = -xd(y)y$ for all $x, y \in N$ (5.18)

Substituting xz for x in equation (5.18), we find that

$$yxzd(y) = -xzd(y)y$$
$$= -x(zd(y)y)$$
$$= -x\big(-yzd(y)\big)$$
$$= -x(-y)zd(y) \text{ for all } x, y, z \in N$$

The last expression reduced to

$yxzd(y) = -x(-y)zd(y)$ for all $x, y, z \in N$ (5.19)

Since, we have $-yxzd(y) = (-y)xzd(y)$,

The equation (5.19) becomes

$(-y)xzd(y) = x(-y)zd(y)$ for all $x, y, z \in N$ (5.20)

Taking $-y$ instead of y in equation (5.20) we obtain

$yxzd(-y) = xyzd(-y)$ for all $x, y, z \in N$.

So that, it reduces to

$(yx - xy)zd(-y) = 0$ and therefore we get

$[y, x]Nd(-y) = \{0\}$ for all $x, y \in N$ (5.21)

By primeness, equation (5.19) assures that for each $y \in N$, either $y \in Z(N)$ or $d(-y) = 0$. Accordingly,

$d(y) = 0$ or $y \in Z(N)$ for all $x, y \in N$ (5.22)

Since equation (5.22) is the same as equation (5.13), $y \in Z(N)$ arguing as $y \in Z(N)$ also implies that $d(y) \in Z(N)$ and equation (5.22) forces $d(y) \in Z(N)$ for all $y \in Z(N)$, which gives $d(N) \subseteq Z(N)$. Hence N is commutative ring by Theorem 4.2.2,

$$d(x)y + yd(x) = xy + yx,$$
$$2d(x)y = 2xy \text{ for all } x, y \in N.$$

Therefore equation (5.16) assures that $d(x)y = xy$ for all $x, y \in N$ and replacing x by xt, This implies that

$$d(xt)y = xty \text{ for all } x, t, y \in N.$$

Since, $t \in N$ implies that $d(t) \in N$, then we have

$$d(xt)y = xty$$

$$\big(d(x)t + xd(t)\big)y = xty$$

Since $d(x)ty + xd(t)y = xty$ for all $x, y, t \in N$, since $d(t) = t$, for any $t \in N$. By using properties

$$d(x)ty = 0 \text{ for all } x, t, y \in N.$$

Therefore we get,

$$d(x)Ny = \{0\} \text{ for all } x, y \in N.$$

By primeness of N and $d \neq 0$, we conclude that $y = 0$ for all $y \in N$, which yields N is zero which is a contradiction. Thus condition in equation (5.16) holds only that of d is zero derivation.

Khan and Madugu in (2017) proved Theorem 5.1.4 and Theorem 5.1.5, in setting of a certain appropriate non-negative integers on derivation of zero symmetric right prime near-rings.

Theorem 5.1.4 (Khan and Madugu, 2017) Let N be a prime near-ring and there exist non-negative integers p and q. If N admits a non-zero derivation d such that
$d(xy - yx) \pm x^p(xy + yx)x^q = 0$ for all $x, y \in N$, then N is a commutative ring.

Proof. We have

$$d([x, y]) = x^p(xy + yx)x^q \text{ for all } x, y \in N. \tag{5.23}$$

Taking y by yx in equation (5.23) and using the properties of commutator (i) and (ii), we find that

$$d([x, y]x) = x^p(xy + yx)x^{q+1} \text{ for all } x, y \in N. \tag{5.24}$$

From the left side of equation (5.24), we get

$$d([x, y]x) = d([x, y])x + [x, y]d(x) \tag{5.25}$$

Combining equation (5.23) and (5.24) in equation (5.25), we obtain the result

$$x^p(xy + yx)x^{q+1} = x^p(xy + yx)x^{q+1} + [x, y]d(x).$$

This implies that

$$[x, y]d(x) = 0 \text{ for all } x, y \in N$$

Replacing y by zy, we have

$$[x, zy]d(x) = [x, z]yd(x) = 0 \text{ for all } x, y, z \in N \tag{5.26}$$

This implies that

$[x,z]Nd(x) = 0$ for all $x, y, z \in N$ (5.27)

In view of primeness of N, equation (5.27) yields that for each $x \in N$,

$d(x) = 0$ or $d(x) \in Z(N)$ (5.28)

Now we have if $x \in Z(N)$, then $d(x) \in Z(N)$.

Hence, equation (5.28) forces that for all $x \in Z(N)$, then $d(x) \in Z(N)$, therefore $d(N) \subseteq Z(N)$. It follows from Theorem 4.2.2 that N is commutative ring.

Next, we show the second conditions

$d([x,y]) = -x^p(xy + yx)x^q$ for all $x, y \in N$. (5.29)

Replacing y by yx and using the properties of commutator (i) and (ii), we have

$d([x,y]x) = -x^p(xy + yx)x^{q+1}$ (5.30)

From the left sides of equation (5.30), we have

$d([x,y]x) = d([x,y])x + [x,y]d(x)$ (5.31)

Use the obtained results of equation (5.29) and (5.30) in equation (5.31) to get

$$-x^p(xy - yx)x^{q+1} = -x^p(xy - yx)x^{q+1} + [x,y]d(x)$$

This gives

$$[x,y]d(x) = 0 \text{ for all } x, y \in N.$$

Replacing y by zy, we have for all $x, y, z \in N$, whch implies

$$[x,zy]d(x) = [x,z]yd(x) = 0 \text{ for all } x, y, z \in N,$$

which implies that

$[x,z]Nd(x) = 0$ for all $x, y \in N$ (5.32)

In view of primeness of a near ring N, equation (5.32) yields that for each $x \in N$,

$d(x) = 0$ or $x \in Z(N)$ (5.33)

So, we know that if $x \in Z(N)$, then $d(x) \in Z(N)$. Hence, equation (5.33) forces that for all $x \in N$ implies $d(x) \in Z(N)$. It follows that $d(N) \subseteq Z(N)$, N is commutative ring.

Theorem 5.1.5 (Khan and Madugu, 2017) Let N be a prime near-ring and there exist non-negative integers p and q. If N admits a non-zero derivation d such that

$d(xy + yx) \pm y^p(xy - yx)y^q = 0$ for all $x, y \in N$, then N is commutative ring.

Proof. We have

$d(xy + yx) = y^p(xy - yx)y^q$ for all $x, y \in N$ (5.34)

Substituting x by xy in equation (5.34) and using properties of commutators (i) and (ii), we find that

$d(xy+yx) = y^p(xy-yx)y^{q+1}$ for all $x, y \in N$ (5.35)

By definition of derivation d, we have

$d\big((xoy)y\big) = d(xoy)y + xoyd(y)$ for all $x, y \in N$ (5.36)

Substituting equation (5.34) and (5.35) in equation (5.36), we get

$$y^p(xy-yx)y^{q+1} = y^p(xy-yx)y^{q+1} + xoyd(y).$$

This implies that $xoyd(y) = 0$ for all $x, y \in N$, since

$yxd(y) = -xyd(y)$ for all $x, y \in N$ (5.37)

Replace x by in tx equation (5.37) and use equation (5.37) to obtain

$$\begin{aligned} ytxd(y) &= -txyd(y) \\ &= (-t)\big(-yxd(y)\big) \\ &= (-t)(-y)xd(y) \text{ for all } x, y, t \in N \text{ or} \\ \big(yt-(t)(-y)\big)xd(y) &= 0 \text{ for all } x, y, t \in N. \end{aligned}$$

Taking y by $-y$, then we have

$(-yt+ty)xd(-y) = 0$ for all $x, y, t \in N$.

Therefore $-[t, y]xd(-y) = 0$ for all $x, y, t \in N$.

This implies that

$[t, y]xd(y) = 0$ for all $x, y, t \in N$. (5.38)

Thus, we have

$$[t, y]Nd(y) = 0 \text{ for all } x, y, t \in N,$$

Since N is prime, we have for each $y \in N$,

$d(y) = 0$ or $y \in Z(N)$ (5.39)

We know that if $y \in Z(N)$, then $d(y) \in Z(N)$. Hence equation (5.39) forces that for $y \in Z(N)$, implies that $d(y) \in Z(N)$. Thus by equation (5.39), N is a commutative ring.

Assume that the next hypotheses

$d(xoy) = -y^p(xy-yx)y^q$ for any $x, y \in N$ (5.40)

Putting x by xy in equation (5.40) and using the properties of commutator (iii), we obtain

$d\big((xoy)y\big) = -y^p(xy-yx)y^{q+1}$ for all $x, y \in N$ (5.41)

By definition of derivation d, we have

$d\big((xoy)y\big) = d(xoy)y + (xoy)d(y)$ for all $x, y \in N$ (5.42)

Using equation (5.40) and (5.41) in equation (5.42) to get

$-y^p(xy - yx)y^{q+1} = -y^p(xy - yx)y^{q+1} + (xoy)d(y)$ (5.43)

This implies that $(xoy)d(y) = 0$ for all $x, y \in N$.

$yxd(y) = -xyd(y)$ for all $x, y \in N$ (5.44)

But equation (5.44) is the same as equation (5.37), arguing as in the above proof of Theorem 5.1.4, we assure that N is a commutative ring.

The following example shows that the primeness hypothesis in Theorems 5.1.4 and Theorem 5.1.5 is necessary even in the case of arbitrary rings.

Example 5.3 (Khan and Madugu, 2017) Let R be a non-commutative ring and $N = \left\{\begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & \beta \\ 0 & 0 & \gamma \end{pmatrix} \Big/ \alpha, \beta, \gamma \in R\right\}$. Define a map $d: N \to N$ by $d\begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & \beta \\ 0 & 0 & \gamma \end{pmatrix} = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, one can easily check that, d is a non-zero derivation on N. Let $A = \begin{pmatrix} 0 & 0 & \alpha_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta_1 \\ 0 & 0 & 0 \end{pmatrix}$, where $A \neq 0$ and $B \neq 0$. Then $ANB = \{0\}$, which shows that N is not prime. Furthermore, d satisfies the condition $d([A, B]) = [A, B]$ and $d([A, B]) = AoB$ or $d(AoB) = [A, B]$ for all A, B in N, but the primeness hypotheses not satisfied. Hence matrix multiplication is not commutative. Therefore N is a non-commutative ring.

5.2. Commuting Condition on Derivation in Prime Near-Rings

In this section we investigate commuting conditions of derivations satisfying certain differential identities on 3-prime near-rings. In this case derivation satisfying certain differential identities on 3-prime near-rings and 2-torsion free conditions facilitate near-ring is commutative ring. Bell, *et al.*, (2012) and Ashraf, *et al.*, (2014) proved some theorems of derivations on 3-prime near-rings satisfying certain differential identities and more precisely, we have conditions which preserve the Lie product $[x, y]$ and the Jordan products xoy in the derivation on near-rings.

Definition 5.2.1 (Bell and Mason, 1992) A mapping $d: N \to N$ is called a commuting derivation if $[d(x), x] = 0$ for all x in N.

The following Theorem 5.2.1 and Theorem 5.2.2 prove that left prime near-ring admitting a nonzero commuting derivation d on N, show N is commutative ring.

Theorem 5.2.1 (Bell, *et al.*, 2012) Let N be a 2-torsion free 3-prime near ring. If N admits a nonzero derivation d such that $[d(x), y] = [x, d(y)]$ for all $x, y \in N$, then N is a commutative ring.

Proof. We are given that

$[d(x), y] = [x, d(y)]$ for all $x, y \in N$ (5.45)

Replacing x by $d(y)x$ in equation (5.43), we obtain

$$[d(d(y)x), y] = [d(y)x, d(y)]$$
$$= d(y)[x, d(y)],$$

From equation (5.45), we get

$$[d(d(y)x), y] = d(y)[d(x), y] \text{ for all } x, y \in N,$$

So that

$d(d(y)x)y - yd(d(y)x) = d(y)d(x)y - d(y)yd(x)$ for all $x, y \in N$ (5.46)

In view of Lemma 4.2.1, equation (5.46) implies that

$$d(y)d(x)y + d^2(y)xy - yd^2(y)x - yd(y)d(x) = d(y)d(x)y - d(y)yd(x)$$

Since $yd(y) = d(y)y$ by equation (5.45), the last equation reduces to

$d^2(y)xy = yd^2(y)x$ for all $x, y \in N$ (5.47)

Replacing x by tx in equation (5.47), we get

$$d^2(y)txy = yd^2(y)tx$$
$$= d^2(y)tyx \text{ for all } x, y, t \in N,$$

This leads to

$$d^2(y)N[x, y] = \{0\} \text{ for all } x, y \in N$$

Since N is 3-prime, we conclude that for each $y \in N$.

$$d^2(y) = 0 \text{ or } y \in Z(N),$$

And substituting $d(y)$ for y in equation (5.45) gives

$$[d(x), d(y)] = 0 \text{ for all } x, y \in N$$

Thus N is a commutative ring by Theorem 4.2.3.

Theorem 5.2.2 (Bell, *et al.*, 2012) Let N be a 2-torsion free 3-prime near ring. If N admits a nonzero derivation d satisfying $[d(x), y] = [d(x), d(y)]$ for all $x, y \in N$, then N is a commutative ring.

Proof. We are given that

$$[d(x), y] = [d(x), d(y)] \text{ for all } x, y \in N \tag{5.48}$$

Replacing y by $d(x)y$ in equation (5.48), we obtain

$$\begin{aligned}[d(x), d(d(x)y)] &= [d(x), d(x)y] \\ &= d(x)[d(x), y] \\ &= d(x)[d(x), d(y)] \text{ for all } x, y \in N,\end{aligned}$$

So that

$$d(x)d(d(x)y - d(d(x)y)d(x) = d(x)d(x)d(y) - d(x)d(y)d(x) \text{ for all } x, y \in N \tag{5.49}$$

In view of Lemma 4.2.1, equation (5.49) implies that

$$d(x)d(x)d(y) + d(x)d^2(x)y - d^2(x)yd(x) - d(x)d(y)d(x) = d(x)d(x)d(y) - d(x)d(y)d(x) \text{ for all } x, y \in N.$$

And by simplifying this we get

$$d(x)d(x)d(y) - d(x)d(y)d(x) \text{ for all } x, y \in N,$$

The last equation reduces to

$$d(x)d^2(x)y = d^2(x)yd(x) \text{ for all } x, y \in N. \tag{5.50}$$

Replacing y by ty in equation (5.50), we get

$$\begin{aligned}d^2(x)tyd(x) &= d(x)d^2(x)ty \\ &= d^2(x)td(x)y \text{ for all } x, y, t \in N,\end{aligned}$$

which leads to

$$d^2(x)N[d(x), y] = \{0\} \text{ for all } x, y, t \in N.$$

Since N is 3-prime, we conclude that for each $x \in N$.

$$d^2(x) = 0 \text{ or } d(x) \in Z(N) \text{ for each } y \in N,$$

So that

$$d^2(x) \in Z(N) \text{ for all } x \in N.$$

Substituting $d(y)$ for y in equation (5.48) gives

$$[d(x), d(y)] = 0, \text{ for all } x, y \in N$$

Thus the results follows from the Theorem 4.2.3, N is commutative ring.

Bell, *et al.*, (2012) proved that differential identities involving ant-commutators instead of commutators are arrive to obtain the derivation on left prime near-ring. We now consider from Theorem 5.2.3 and Theorem 5.2.4 differential identities involving anti-commutators instead of commutators.

Theorem 5.2.3 (Bell, *et al*, 2012) If N is a 2-torsion free 3-prime near ring, then N admits no nonzero derivation d such that $d(x)od(y) = 0$, for all $x, y \in N$.

Proof. Suppose d is a nonzero derivation such that

$d(x)d(y) + d(y)d(x) = 0$ for all $x, y \in N$ (5.51)

Then, $d(x) = d(y)$ we get

$$d(x)od(x) = 2d(x)^2$$
$$= 0, \text{ so that } d(x)^2 = 0 \text{ for all } x, y \in N.$$

Thus,

$$d\big(d(x)d(x)\big)y = 0 = (d^2(x)d(x) + d(x)d^2(x))y$$
$$= d^2(x)d(x)y + d(x)d^2(x)y \text{ for all } x, y \in N,$$

And in particular,

$d^2(x)d(x)d(y) = -d(x)d^2(x)d(y)$ for all $x, y \in N$ (5.52)

Now replacing y by $d(x)y$ in equation (5.51), there by obtaining

$$d(x)(d^2(x)y + d(x)d(y) + (d(x)d(y) + d^2(x)y)d(x) = 0$$

Which it can be rewritten as

$d(x)d^2(x)y + d(x)\big(d(x)d(y) + d(y)d(x)\big) + d^2(x)yd(x) = 0$ for all $x, y \in N$ (5.53)

In light of equation (5.51), the given equation (5.53) yields to

$$d^2(x)yd(x) = -d(x)d^2(x)y \text{ for all } x, y \in N,$$

So that we have

$$d^2(x)d(y)d(x) = -d(x)d^2(x)d(y) \text{ and}$$

Hence

$-d^2(x)d(x)d(y) = -d(x)d^2(x)d(y)$ for all $x, y \in N$. (5.54)

In view of equation (5.52) and (5.54), now we have

$d(x)d^2(x)d(y) = 0$ for all $x, y \in N$ and if $d(y) \in N$, we have

$$d^2(x)d(y)d(x) = 0 \text{ for all } x, y \in N,$$

It now follows from equation (5.54) that $d^2(x)Nd(x) = \{0\}$ for all $x \in N$ and by 3-primeness of N, then $d^2(x) = 0$ for all $x \in N$.

Therefore by Lemma 4.2.3(iv), we get $d = 0$ which contradicts our original assumption that $d \neq 0$. Hence the conditions equation (5.51) holds only for zero derivation on N.

The following example proves that the primeness hypothesis in Theorem 5.2.1, Theorem 5.2.2 and Theorem 5.3.3 is necessary to say N is commutative ring.

Example 5.4 (Bell, *et al*, 2012) Let S be a noncommutative 2-torsion free left near ring. Let us consider $N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} / x, y, z \in S \right\}$ and $d \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix} = \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. It is straight forward to check that N is a 2-torsion free non 3-prime left near-ring with $Z(N) \neq \{0\}$. Moreover, d is a nonzero derivation of N satisfying the following conditions

(i) $[d(A), B] = [A, d(B)]$,

(ii) $[d(A), B] = [\, d(A), d(B)]$,

(iii) $d(A)od(B) = 0$ for all $A, B \in N$ and N is not a commutative ring.

Bell, *et al.*, (2012, Theorem 2.5) proved that 2-torsion free left prime near-ring, if N admits nonzero derivation, then $Z(N) = \{0\}$ and N is not ring.

Theorem 5.2.4 (Bell, *et al*, 2012) Let N be a 2-torsion free 3-prime near-ring. If N admits a nonzero derivation d such that $d(x)od(y) = xoy$ for all $x, y \in N$, then $Z(N) = \{0\}$ and N is not a ring.

Proof. Assume that from the given

$$d(x)od(y) = xoy \text{ for all } x, y \in N \tag{5.55}$$

Replacing y by yz in equation (5.55), where $z \in Z(N)$, we get

$$\begin{aligned} d(x)od(yz) &= xo(yz) \\ &= (xoy)z \\ &= \big(d(x)od(y)\big)z \text{ for all } x, y \in N \end{aligned}$$

And therefore

$$d(x)\big(d(y)z + yd(z)\big) + (yd(z) + d(y)z)d(x) = d(x)d(y)z + d(y)d(x)z \tag{5.56}$$

Using Lemma 4.2.1, equation (5.56) becomes

$d(x)yd(z) + yd(z)d(x) = 0$ for all $x, y \in N, z \in Z(N)$ (5.57)

Since we have $d(z) \in Z(N)$ and from equation (5.57) it follows that

$\big(d(x)y + yd(x)\big)Nd(z) = 0$ for all $x, y \in N, z \in Z(N)$ (5.58)

From the equation (5.58), we have that $d\big(Z(N)\big) = 0$ or $d(x)y + xd(y) = 0$ for all $x, y \in N$.

The latter case, $d(x)d(y) + d(y)d(x) = 0$ for all $x, y \in N$, so this case cannot occur by Theorem 5.2.3.

If we take $d\big(Z(N)\big) = \{0\}$, then for any $y \in Z(N)$ equation (5.55) implies that

$$\begin{aligned} 2xy &= xoy \\ &= d(x)od(y) \\ &= 0, \text{ for all } x \in N. \end{aligned}$$

Since N is 2-torsion free, then $xy = 0$ for all $x \in N$.

Hence $yNy = 0$ for all $y \in Z(N)$ and the 3-primeness of N yields $Z(N) = \{0\}$.

Suppose N ring. Then from $2d(x)^2 = 2x^2$, we have $d(x)^2 = x^2$ for all $x \in N$.

Now, $d(x)od(x^2) = 2x^3$

Hence,

$$d(x)\big(d(x)x + xd(x)\big) + (d(x)x + xd(x))d(x) = 2x^3 \text{ for all } x \in N,$$

So that

$$d(x)^2x + d(x)xd(x) + d(x)xd(x) + x\,d(x)^2 = 2x^3 \text{ for all } x \in N,$$

Accordingly,

$$x^3 + 2d(x)xd(x) + x^3 = 2x^3 \text{ for all } x \in N,$$

So $2d(x)xd(x) = 0$ and therefore

$$d(x)xd(x) = 0 \text{ for all } x \in N,$$

Pre-multiplying and post-multiplying by $d(x)$ gives $d(x)^2xd(x)^2 = 0$, $x^5 = 0$ for all $x \in N$.

But this cannot happen, for by considering the result of a prime ring cannot be nil of bounded index. Thus, our assumption that N is ring yields a contradiction. Therefore N is not ring.

The following results follow from theorem 5.2.4.

Corollary 5.1 (Bell, *et al*, 2012) Let N be a 2-torsion free 3-prime near-ring with $Z(N) \neq \{0\}$. Then N admits no nonzero derivation d such that $d(x)od(y) = xoy$ for all $x, y \in N$.

The following example proves that the primeness hypothesis in Theorem 5.2.4 cannot be omitted.

Example 5.5 (Bell, *et al*, 2012) Let S be a non commutative 2-torsion free left near-ring. Let us consider $N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} / x, y, z \in S \right\}$ and a map defined by $d: N \to N$, then $d\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix}$ for all $x, y, z \in S$. Therefore d is derivation on N, but N is neither a 3-prime nor ring. For instant, let $x = \begin{pmatrix} 0 & 2x_1 & x_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $y = \begin{pmatrix} 0 & 2y_1 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ for all x_1, $x_2, y_1, y_2 \in S$, it is straight forward d is derivation on N. We have $xNy = \{0\}$, but $x \neq 0$ and $y \neq 0$. From the above observations $d(x)od(y) = xoy$ for all $x, y \in N$, then $Z(N) \neq \{0\}$ and N is neither a prime nor ring.

We consider from Theorem 5.2.5, a zero symmetric prime right near-rings with derivation satisfying the following conditions.

Theorem 5.2.5 (Ashraf *et al.*, 2014) Let N be a right prime near-ring which admits a nonzero derivation d. Then the following assertions are equivalent

(i) $d([x, y]) = [d(x), y]$ for all $x, y \in N$,

(ii) $[d(x), y] = [x, y]$ for all $x, y \in N$,

(iii) N is a commutative ring.

Proof. First we show (i) ⇒ (iii).

Assume that

$$d([x, y]) = [d(x), y] \text{ for all } x, y \in N \tag{5.59}$$

Replacing y by yx in equation (5.59) we get

$$[d(x), yx] = d([x, y]x) \text{ for all } x, y \in N \tag{5.60}$$

By definition of derivation d, equation (5.60) reduced to

$xyd(x) = yd(x)x$ for all $x, y \in N$ (5.61)

Substituting yz for y in equation (5.61) where $z \in N$, we obtain $[x, y]zd(x) = 0$ which leads to

$[x, y]Nd(x) = \{0\}$ for all $x, y \in N$. (5.62)

Since N is a prime, equation (5.62) yields

$d(x) = 0$ or $[x, y] = 0$ for all $x, y \in N$ (5.63)

Then equation (5.63) it follows that for each fixed $x \in N$, then we have

$d(x) = 0$ or $d(x) \in N$ (5.64)

But $x \in Z(N)$ also implies that $d(x) \in Z(N)$ and equation (5.64) forces $d(x) \in Z(N)$ for all $x \in N$, hence $d(N) \subseteq Z(N)$, we conclude that N is a commutative ring.

Let we show (ii) ⇒ (iii), from the hypothesis

$[d(x), y] = [x, y]$ for all $x, y \in N$ (5.65)

Replacing x by in xy equation (5.65) and by using the properties of commutators

$$[xy, y] = [x, y]y,$$

Then we get

$[d(xy), y] = [x, y]y = [d(x), y]y$ for all $x, y \in N$

In view of Lemma 4.2.1(i), the last equation can be rewritten as

$$d(x)y^2 + xd(y)y - yxd(y) - yd(x)y = d(x)y^2 - yd(x)y,$$

So that

$xd(y)y = yxd(y)$ for all $x, y \in N$ (5.66)

Substituting yz for y in equation (5.66), where $z \in N$, we obtain that $[x, y]zd(y) = 0$ which leads to

$[x, y]Nd(y) = \{0\}$ for all $x, y \in N$ (5.67)

Since N is a prime, equation (5.67) yield

$d(y) = 0$ or $[x, y] = 0$ for all $x, y \in N$ (5.68)

Then equation (5.68) it follow that for a fixed $y \in N$, we have

$d(y) = 0$ or $d(y) \in N$ (5.69)

But $y \in Z(N)$ also implies that $d(y) \in Z(N)$ and equation (5.69) forces $d(y) \in Z(N)$ for all $x, y \in N$. Hence $d(N) \subseteq Z(N)$ and by primness, N is commutative ring.

We show that (iii) ⇒ (i).

Suppose N is commutative ring, for all $x, y \in N$, since $xy = yx$.

We need to show that $d([x,y]) = [d(x), y]$, for all $x, y \in N$, the from the left side

$$d([x,y]) = d(xy - yx)$$
$$= d([xy - xy), \text{ since } N \text{ is commutative ring,}$$
$$= d(0) = 0 \text{ for all } x, y \in N \qquad (5.70)$$

From the right side,

$[d(x), y] = d(x)y - yd(x)$, $d(x) \in N$, since N is commutative ring $d(x)y = yd(x)$.

$$[d(x), y] = d(x)y - d(x)y$$
$$= 0 \text{ for all } x, y \in N \qquad (5.71)$$

From equation (5.70) and (5.71) we conclude that $d([x,y]) = [d(x), y]$ for all $x, y \in N$.

We show that (iii) ⇒ (ii).

Suppose N is commutative ring, we need to show that

$$[d(x), y] = [x, y] \text{ for all } x, y \in N.$$

From the left side,

$[d(x), y] = d(x)y - yd(x)$ for all $x, y \in N$, $d(x), y \in N$, then we get $d(x)y = yd(x)$, N is commutative ring we have

$$[d(x), y] = d(x)y - d(x)y$$
$$= 0 \text{ for all } x, y \in N. \qquad (5.72)$$

From the right side

$$[x, y] = xy - yx, \text{ for all } x, y \in N.$$

From this

$$[x, y] = xy - xy \text{ for all } x, y \in N, \text{ since } N \text{ is commutative ring } xy = yx.$$

$[x, y] = xy - xy$ for all $x, y \in N$ (5.73)

From equation (5.72) and equation (5.73), we conclude that $[d(x), y] = [x, y]$ for all $x, y \in N$.

5.3. Centralizing Derivation on Prime Near-Rings

Bell and Mason (1987) defined centralizing derivation of a near-ring. Inspired by the study of centralizing derivation in rings, they studied the concepts of centralizing derivations in near rings. Motivated by their study recently, Ashraf *et at.*, (2014) proved a prime near-ring which

admits a nonzero centralizing derivation d on N. In this section it is shown that zero symmetric prime right near-rings with derivations satisfying certain identities are commutative ring.

Definition 5.3.1 (Bell and Mason, 1992) A mapping $d: N \to N$ is said to be centralizing derivation on a near-ring N if $[d(x), x] \in Z(N)$, holds for all x in N.

We facilitate our discussion with the following Lemma 5.1 which is required for developing the proof of the following theorems.

Lemma 5.1 (Ashraf, *et al.*, 2014) Let N be a 2-torsion free right prime near-ring. If N admits a nonzero derivation d such that $d([x, y]) = 0$ for all $x, y \in N$, then N is a commutative ring.

Proof: Let we have given that

$d([x, y]) = 0$ for all $x, y \in N$ (5.74)

From the equation (5.74), we get

$d(xy - yx) = d(xy) + \big(-d(yx)\big) = 0$ for all $x, y \in N$ (5.75)

By comparing equation (5.74) and (5.75), we get $d(xy) = d(yx)$ for all $x, y \in N$.

Now,

$d(x)y + xd(yyd(x) = d(y)x + yd(x)$ for all $x, y \in N$ (5.76)

Therefore, equation (5.76) argue that N is a commutative ring.

Ashraf, *et al.*, (2014) proved Theorem 5.3.1 and Theorem 5.3.2 by using primeness condition on a nonzero derivation d for which $d(N) \subseteq Z(N)$, then the following condition hold.

Theorem 5.3.1 (Ashraf *et al.*, 2014) Let N be a 2-torsion free prime near-ring which admits a nonzero derivation d. Then the following assertions are equivalent

(i) $d([x, y]) \in Z(N)$for all$x, y \in N$.

(ii) N is a commutative ring.

Proof. We show first (i)⇒ (ii)

Suppose we are given that

$d([x, y]) \in Z(N)$ for all $x, y \in N$ (5.77)

Case (i) If we take $Z(N) = \{0\}$, it follows $d([x, y]) = 0$ for all $x, y \in N$. By Lemma 5.1, we conclude that N is a commutative ring.

Case (ii) Let $Z(N) \neq \{0\}$, replacing y by in yz equation (5.77), where $z \in Z(N)$, $d([x,y]z) \in Z(N)$. Then we get

$d([x,y]z) + [x,y]d(z) \in Z(N)$ for all $x, y \in N$, $z \in Z(N)$. (5.78)

Using equation (5.77) together with Lemma 4.2.1(i) and equation (5.78) implies

$$[x,y]d(z) \in Z(N) \text{ for all } x, y \in N, z \in Z(N).$$

Accordingly,

$$\big[[x,y]d(z), t\big] = \big[[x,y], t\big]d(z) = 0, \text{ for all } t \in N,$$

Thus

$\big[[x,y], t\big]Nd(z) = \{0\}$ for all $x, y, t \in N, z \in Z(N)$ (5.79)

Using the primeness of N, from equation (5.79) it follows that $d\big(Z(N)\big) = 0$ or $\big[[x,y], t\big] = 0$ for all $x, y, t \in N$.

Assume that $\big[[x,y], t\big] = 0$ for all $x, y, t \in N$, substituting yx for y we get $\big[[x,y], t\big] = 0$ and therefore $[x,y][x,t] = 0$ for all $x, y, t \in N$. As $[x,y] \in Z(N)$,

Hence we get

$[x,y]N[x,y] = \{0\}$ for all $x, y \in N$. (5.80)

In light of the primeness of N, the equation (5.80) shows that $[x,y] = 0$ and hence $x \in Z(N)$.

Accordingly,

$d(x) \in Z(N)$ for all $x, y \in N$. (5.81)

Once again using Lemma 5.1, we get the required result. On the other hand, if

$d\big(Z(N)\big) = \{0\}$, then $d^2([x,y]) = 0$ for all $x, y \in N$ (5.82)

Replacing y by yx, we have $0 = d^2([x,yx])$

$$= d^2([x,y])x + 2d([x,y])d(x) + [x,y]d^2(x) \text{ for all } x, y \in N.$$

Then we have, $d^2([x,yx]) = d^2([x,y])x$

Hence

$$2d([x,y])d(x) + [x,y]d^2(x) = 0 \text{ for all } x, y \in N. \quad (5.83)$$

Taking $[u,v]$ instead of in equation (5.83) and using the hypothesis 2-torsion freeness, we have

$d([u,v])Nd\big(\big[[u,v], y\big]\big) = \{0\}$ for all $u, v, y \in N$. (5.84)

Since N is 3-prime, then equation (5.84) shows that

$d([u,v]) = 0$ or $d\big(\big[[u,v], y\big]\big) = 0$ for all. $u, v, y \in N$ (5.85)

Therefore in view of equation (5.78), equation (5.85) implies that

$d([u,v]) = 0$ or $[u,v]d(y) = d(y)[u,v]$ for all. $u,v,y \in N$ (5.86)

If there are two elements $u_o, v_o \in N$ such that

$[u_o,v_o]d(y) = d(y)[u_o\ ,v_o]$ for all $y \in N$. (5.87)

Replacing y by $[u_o,v_o]t$ in the first term of equation (5.87) and using Lemma 4.2.1(i), we have

$$[u_o,\ v_o]d([u_o,\ v_o]t) = [u_o,\ v_o]d([u_o,\ v_o])t + [u_o,v_o]^2 d(t)$$
$$= d([u_o,\ v_o])\ [u_o,\ v_o]t + [u_o,v_o]^2 d(t)$$

Taking $[u_o,\ v_o]t$ instead of y in the second term of equation (5.87) and using equation (5.84), we get

$d([u_o,v_o]t)[\ u_o,v_o] = d([u_o,v_o]t[u_o\ ,v_o] + [u_o,v_o]^2 d(t)$ (5.88)

Comparing the last two equations (5.87) and (5.88), we get

$$d([u_o,v_o]N[[u_o\ ,v_o],t] = \{0\} \text{ for all } t \in N.$$

Since N is 3-prime, we get that

$$d([u_o,\ v_o]) = 0 \text{ or } [[u_o,\ v_o],t] = 0 \text{ for all } t \in N.$$

Taking equation (5.87), into account, equation (5.86) implies that

$d([u,\ v]) = 0$ or $[[u,\ v],t] = 0$ for all $t,u,v \in N$. (5.89)

If there are two elements u_1,$v_1 \in N$ such that $[[u_1,\ v_1],t] = 0$ for all $t \in N$, then

$$[u_1,\ v_1] \in Z(N).$$

By the hypothesis, we get $d([u_1,\ v_1]) = 0$, from equation (5.89) and the last expression, we get

$$d([u,v]) = 0 \text{ for all } u,v \in N.$$

Hence, N is a commutative ring by Lemma 5.1.

Next, we show the converse or (ii) ⇒ (ii)

Suppose N is commutative ring, then for some $x \in Z(N)$ implies that $xy = yx$, since derivation d on N, we have $d(xy) = d(yx)$ for all $x,y \in N$ or

$d(xy - yx) = d(x)y + xd(y) - d(y)x - yd(x) = 0$ for all $x,y \in N$ (5.90)

Therefore from equation (5.90), we get $d([x,y]) = 0$. Now d is a zero derivation $Z(N) \neq \emptyset$, this shows that $0 \in Z(N)$. It follow from Lemma 5.1, then equation (5.77) assure that $d([x,y]) \in Z(N)$ for all $x,y \in N$.

Ashraf, *et al.*, (2014) proved Theorem 5.3.2 by using primeness condition on a nonzero derivation d for which $d(N) \subseteq d(N)$, show the equivalent condition hold.

Theorem 5.3.2 (Ashraf *et al.*, 2014) Let N be a prime near-ring which admits a nonzero derivation d. Then the following assertions are equivalent

(i) $[d(x), y] \in Z(N)$ for all $x, y \in N$,

(ii) N is a commutative ring.

Proof. We first show that (i) ⇒ (ii).

Assume that

$$[d(x), y] \in Z(N) \text{ for all } x, y \in N. \tag{5.91}$$

Hence, by using Lemma 4.2.1, for any $t \in N$, the we get

$$\begin{aligned} [d(x), y]t &= d(x)yt - tyd(x) \\ &= \big[[d(x), y], t\big] \\ &= 0, \text{ for all } x, y, t \in N. \end{aligned} \tag{5.92}$$

Therefore $\big[[d(x), y], t\big] = 0$ and replacing y by $yd(x)$ in the equation (5.92) we find that

$$\big[[d(x), y]d(x), t\big] = 0 \text{ for all } x, y, t \in N. \tag{5.93}$$

In view of equation (5.91), equation (5.93) assures that

$$[d(x), y]t[d(x), y] = \{0\} \text{ for all } x, y, t \in N,$$

Since for any $t \in Z(N)$, then we have

$$[d(x), y]N[d(x), y] = \{0\} \text{ for all } x, y \in N. \tag{5.94}$$

By primeness of N, the equation (5.94) shows that

$$[d(x), y] = 0 \text{ for all } x, y \in N.$$

Hence, $d(N) \subseteq Z(N)$ and by using Lemma 5.1 assures that N is a commutative ring.

Next, we show (ii) ⇒ (i).

Suppose N is commutative ring, then for any $x, y \in N$ implies that $d(x)y = yd(x)$, since derivation d on N, we have

$$d(x)y - yd(x) = 0 \text{ for all } x, y \in N \tag{5.95}$$

Therefore from equation (5.95), we get $[d(x), y] = 0$ for all $x, y \in N$. Now N is commutative ring, we have $d(x)y - yd(x) = 0$ follow that $d(x) \in Z(N)$. It follow from Lemma 5.1, then equation (5.91) assure that $[d(x), y] \in Z(N)$ for all $x, y \in N$.

We now consider differential identities involving anti-commutators instead of commutators. We obtain the following results:

Theorem 5.3.3 (Ashraf *et al.*, 2014) Let N be a 2-torsion free prime near-ring which admits a nonzero derivation d. Then the following assertions are equivalent

(i) $d(xoy) \in Z(N)$ for all $x, y \in N$,

(ii) N is a commutative ring

Proof. We show (i) ⇒ (ii).

Suppose that

$d(xoy) \in Z(N)$ for all $x, y \in N$ (5.96)

Case 1: If we take $Z(N) = \{0\}$, then $d(xoy) = 0$ and replacing y by yx in equation (5.96) we obtain and thus

$xyd(x) = -yxd(x)$ for all $x, y \in N$ (5.97)

Substituting yz for y in equation (5.97), we have

$$\begin{aligned} xyzd(x) &= -yzxd(x) \\ &= -y\big(-xzd(x)\big) \\ &= -y(-x)zd(x) \text{ for all } x, y \in N. \end{aligned}$$

This means that

$xyzd(x) = -y(-x)zd(x)$ for all $x, y, z \in N$ (5.98)

Since we have

$$-xyzd(x) = (-x)yzd(x),$$

Then equation (5.98) becomes

$(-x)yzd(x) = y(-x)zd(x)$ for all $x, y, z \in N$. (5.99)

Then from equation (5.99), we have

$[-x, y]zd(x) = 0$ for all $x, y, z \in N$. (5.100)

Taking $-x$ instead of x in equation (5.100) gives us

$[x, y]zd(-x) = 0$ for all $x, y, z \in N$. (5.101)

Accordingly,

$[x, y]Nd(-x) = \{0\}$ for all $x, y \in N$. (5.102)

Since equation (5.102) is the same as equation (5.19), arguing as in the proof of Theorem 5.1.3, we conclude that N is a commutative ring.

Case 2: If we take $Z(N) \neq \{0\}$, replacing y by yz in equation (5.94), where $z \in Z(N)$, we get

$$d(xoy)z + (xoy)d(z) \in Z(N) \text{ for all } x, y \in N, z \in Z(N). \quad (5.103)$$

Using equation (5.96) together with Lemma 4.3.1(i), equation (5.103) reduces to

$$(xoy)d(z) \in Z(N) \text{ for all } x, y \in N, z \in Z(N). \quad (5.104)$$

Since $d(z) \in Z(N)$, equation (5.104) yields that

$$[(xoy)d(z), t] = [xoy, t]d(z) = 0.$$

So that we have

$$[xoy], t]Nd(z) = \{0\} \text{ for all } x, y \in N, z \in Z(N),$$

By primeness of N, the last equation forces either

$$d\big(Z(N)\big) = \{0\} \text{ or } xoy \in Z(N) \text{ for all } x, y \in N \quad (5.105)$$

Suppose that we take $d\big(Z(N)\big) = \{0\}$. If $0 \neq y \in Z(N)$, Since

$$d(xoy) = d(x)y + d(x)y \in Z(N),$$

Then we have

$$d(d(x)y + d(x)y) = 0$$

And we have

$$\big(d^2(x) + d^2(x)\big)y = 0 \text{ for all } x \in N \quad (5.106)$$

Using the fact that $0 \neq y \in Z(N)$, equation (5.106) leads to $d^2(x) = 0$ for all $x \in N$.

So that $d^2 = 0$ and Lemma 4.2.3(iv) forces that $d = 0$, a contradiction of our assumption.

Accordingly, we have

$$xoy \in Z(N) \text{ for all } x, y \in N.$$

Let we take $0 \neq y \in Z(N)$, from $xoy = y(x + x)$, $x^2oy = y(x^2 + x^2)$ it follows, because of the primeness hypothesis, that

$$x + x \in Z(N),\ x^2 + x^2 \in Z(N) \text{ for all } x \in N$$

Thus

$$\begin{aligned}(x + x)xt &= (x^2 + x^2)t \\ &= t(x^2 + x^2) \\ &= t(x + x)x \\ &= (x + x)tx \text{ for all } x, t \in N\end{aligned}$$

and therefore

$(x + x)N[x, t] = \{0\}$ for all $x, t \in N$. (5.107)

Once again using the primeness hypothesis, equation (5.107) yields $x \in Z(N)$ or $2x = 0$ in which case 2-torsion freeness forces $x = 0$. Consequently, in both the cases we arrive at $x \in Z(N)$ for all $x \in N$. Hence $d(N) \subseteq Z(N)$ and Theorem 4.2.2 assures that N is a commutative ring.

Next, suppose that N is commutative ring, then for some $x \in Z(N)$ implies that$xy = yx$ and $Z(N) = 0$, then $d(xoy) = 0$, for all $x, y \in N$.

Hence, $d(xy + yx) = 2d(xy)$. By 2-torsion freeness $d(xy) = 0$, for all $x, y \in N$.

Therefore $d(N) \subseteq Z(N)$ and equation (5.92) assure that $d(xoy) \in Z(N)$ for all $x, y \in N$.

The following Theorem 5.3.4 prove that 2-torsion free right prime near ring which admits a nonzero derivation d, then the following conditions hold.

Theorem 5.3.4 (Ashraf, *et al.*, 2014) Let N be a 2-torsion free prime near-ring which admits a nonzero derivation d. Then the following assertions are equivalent

(i) $d(x)oy \in Z(N)$ for all $x, y \in N$,

(ii) N is a commutative ring.

Proof. We show that (i) ⇒ (ii).

Suppose that

$d(x)oy \in Z(N)$ for all $x, y \in N$. (5.108)

Case 1: If $Z(N) = \{0\}$, then equation (5.108) reduced to

$d(x)y = -yd(x)$ for all $x, y \in N$. (5.109)

Substituting yz for y in equation (5.109) we obtain

$$\begin{aligned} d(x)yz &= -yzd(x) \\ &= (-y)zd(x) \\ &= (-y)(-d(x)z) \\ &= (-y)d(-x)z \text{ for all } x, y, z \in N, \end{aligned}$$

in such a way,

$\big(d(x)y + yd(-x)\big)z = 0$ for all $x, y, z \in N$. (5.110)

Taking $-x$ instead of x in equation (5.110) we get

$\big(-d(x)y + yd(x)\big)Nz = \{0\}$ for all $x, y, z \in N$. (5.111)

Since N is prime, equation (5.104) forces $d(N) \subseteq Z(N)$ and from Theorem 4.2.2 it follows that N is a commutative ring.

Case 2: Suppose that $Z(N) \neq \{0\}$. If we take $0 \neq z \in Z(N)$, since $d(x)oz \in Z(N)$, we find that

$d(x) + d(x) \in Z(N)$ for all $x \in N$ (5.112)

Moreover, from equation (5.112) it follows that

$$d(x + x)y + yd(x + x) \in Z(N)$$

Equation (5.112), yields that

$\big(d(x + x) + d(x + x)\big)y \in Z(N)$ for all $x, y \in N$

And therefore, for all $t, x, y \in N$ we have

$$\big(d(x + x) + d(x + x)\big)ty = y\big(d(x + x) + d(x + x)\big)t$$
$$= \big(d(x + x) + d(x + x)\big)y \text{ for all } x, t \in N$$

So that

$\big(d(x + x) + d(x + x)\big)N[t, y] = \{0\}$ for all $t, x, y \in N$. (5.113)

In view of the primeness of N, equation (5.113) implies that either $d(x + x) + d(x + x) = 0$ or $[t, y] = 0$ and thus $d = 0$, is a contradiction or $N \subseteq Z(N)$ in which case $d(N) \subseteq Z(N)$ and N is a commutative ring by Theorem 4.2.2..

Next, let N is commutative ring, we have $d(x)oy = d(x)y + yd(x)$ for any $x, y \in N$, if there is a nonzero derivation on N,

$d(x)y + yd(x) = 0$ for any $x, y \in N$. (5.114)

Equation (5.114) is a zero derivation, then

$$d(x)y = -d(x)y \text{ for all } x, y \in N.$$

Then, we get $d(x)y = 0$, it follows that $0 \in Z(N)$.

Hence we assure that from equation (5.108), $d(x)oy \in Z(N)$ for all $x, y \in N$.

6. SUMMARY AND CONCLUSION

6.1. Summary

Near-rings are one of the generalized structures of rings. But it is difficult to say always near ring is commutative ring, because it did not satisfy all the properties of commutative ring. Some methods have been needed to describe the structure of near-ring is commutative ring. It is possible by using derivation on near ring satisfying certain identities to say N is commutative ring. Mainly, this project is studied commutativity of near rings with derivations satisfying certain differential identities. The work concerning commutativity of near-rings with derivations stating that the existence of a suitably constrained derivation on a prime near-ring forces the near-ring to be a commutative ring. The ways of commuting conditions on derivation in 2-torsion free zero symmetric prime near-ring N, which shows N is commutative ring. Most of the results are devoted to the study commutativity of prime near-rings satisfying certain conditions involving derivations. It is proved that in different theorems which facilitates our aim of these project to be clear. Finally, some examples are given to show the primeness hypothesis are necessary even in the case of arbitrary rings.

6.2. Conclusion

From the results of the study on derivation in near rings, the concepts were deduced by different authors and near ring with derivations satisfying certain identities are commutative ring. This project, prove that a prime near-ring which admits a nonzero derivation satisfying certain differential identities is a commutative ring. Moreover, commutativity of prime near ring satisfying certain conditions involving derivations have been studied. Finally, from the result of commuting derivation and centralizing derivation, concluded that N is commutative near-ring.

7. REFERENCES

Ashraf, M. and Rehman, N.U. 2002. On commutativity of rings with derivations. *Results in Mathematics*, 42(1-2):3-8.

Ashraf, M. Boua, A. and Raji, A. 2014. On derivations and commutativity in prime near-rings. *Journal of Taibah University for Science*, 8:301–306

Ashraf, M. 1993. On structure and commutativity of certain periodic near rings, *Resultate Mathematics*, 24:1-10.

Ashraf, M. 1995. Structure of certain periodic rings and near rings. *Rend. Sem. Mathematics University Pol. Torino*, 1: 61-67.

Argac, N. and Bell, H.E. 2004. Some results on derivations in near rings and near fields. *Kluwer Dordrecht*, 4:42-46.

Beidar, K.I. Fong, Y. and Wang, X.K. 1996. Posner and Herstein theorems for derivations of 3-prime near-rings. *Communications in algebra*, 24(5):1581-1589.

Beidleman, J.C. 1965. A radical for near-ring modules. *Michigan Mathematics. J*, 12(2): 377-383.

Bell, H.E. 1971. Certain near rings are rings, *J. London Mathematics Society*, 2(4):264-270.

Bell, H.E. 1997. On Derivations in Near-Rings-II, *Kluwer Academic Publishers, Netherlands*, 191–197.

Bell, H.E. and Argac, Nurcan. 2001. Derivations, Product derivations and commutativity in near-rings, *Algebra Colloq*. 81(4):399-407.

Bell, H.E. and Daif, M. N. 1994. On commutativity and strong commutativity preserving maps, *Canada Mathematics Bull*. 37(4):443-447.

Bell, H.E. and Mason, G. 1987. On derivations in near-rings. In *North-Holland Mathematics Studies*, 137:31-35.

Bell, H.E. and Mason, G. 1992. On derivations in near-rings and rings. *Mathematics J. Okayama University*, 34:135-144.

Bell, H.E. and Argac, Nurcan. 2004. Some results on derivation in near rings. *Algebra Colloquium* 8(4):399-407.

Bell, H.E. Boua, A. Oukhtite, L. 2012 on derivations of prime near-rings, *African Diaspora J. Mathematics*, 14(1):65-72.

Bresar, M. and Vukman, J. 1991. Jordan $(\theta - \phi)$-derivations, *Glasnik Matematicki* 26(46):13-17.

Boua, A. and Oukhtite, L. 2011. Derivations on prime near-rings. *International J. Open Problems Computational Mathematics*, 4(2):162-167.

Boua, A. 2012. Some Condition under which prime Near-Rings are Commutative Rings. *International Journal Open Problems Computational Mathematics*, 5(2):7-15.

Clay, J.R. 1964. The Near-Rings on a Finite Cyclic Group. *The American Mathematical Monthly*, 71(1): 47-50.

Daif, M. N. and Bell, H. E. 1992. Remarks on derivations on semiprime ring, *International J. Mathematics Science*, 15:205-206.

Deng, Q. Yenigul, M.S. and Argac, N. 1998. On commutativity of near-rings with derivation, *Mathematics Proc. Royal Irish Academy*, 98(2):217-222.

Dickson, L.E. 1905. Definition of group and a field by independent postulates, *Trans. American Mathematical Society*, 6:198-204.

Frohlich, A. 1985. d — g near rings (II-Representation Theory), *Proc. London Mathematical Society*, 3(8):264-270.

Gerhand, W. 2009. On zero divisors if near-rings, *International Journal of Algebra*, 3(1):21-32.

Ghowsi, H. 2014. A commutativity study for Near-Rings, *Payan Noor University research grant*, 1-2.

Herstein, I. N. 1978. A note on derivations, *Canada Mathematics Bull.* 21:3.

Jemal, M. R. 2008. A study of derivation in near ring, Doctorial dissertation, Aligarh Muslim University.

Ligh, S. and Luh, J. 1976. Some commutativity theorems for rings and near-rings, *Acta Mathematics Science Hunger*, 28:19-23.

Khan, Moharram A. and Madugu, Abdu. 2017. Some Commutativity Theorems for Prime Near-rings Involving Derivations. *Journal of Advances in Mathematics and Computer Science*, 25(6):1-9.

Posner, E. C. 1957. Derivations in prime rings, *Proc. American Mathematical Society*, 8:1093-1100.

Pilz, G. 1983. *Near-Rings*, Revised Edition, North-Holland, Amsterdam.

Wang, X.K. 1994. Derivations in prime near-rings. *Proceedings of the American Mathematical Society*, 121(2):361-366.

Yihun, Shang. 2011. A study of derivations in prime near-rings, *Mathematical Balkanica*, 25:413-418.

Yilun, Shang. 2015. A Note on the Commutativity of Prime Near-rings, *Algebra Colloquium*, 22(3):361-366.